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Short Communication

Some remarks on nonlinear vibrations of ideal and nonideal slewing flexible structures

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Abstract

This work investigates a mathematical model for flexible slewing structures assuming nonlinear curvature (including cubic terms) and two different approaches for the interaction actuator–structure: ideal (structure dynamics do not affect actuator dynamics) and nonideal (structure dynamics do affect actuator dynamics).

1. Introduction

When a mathematical model for a system with an energy source (some kind of actuator) is under development, two different assumptions can be made regarding the interaction among the system being excited and the source of the excitation. It can be considered that the energy source is affecting the response of the excited system but the response of the excited system is not affecting the behavior of the energy source or it can be considered that the response of the excited system is affecting the behavior (response) of the energy source. In the first case, the whole system is called an ideal system and, in the second case, it is called a nonideal system [1]. For a complete review on nonideal vibrating systems, see Ref. [2].

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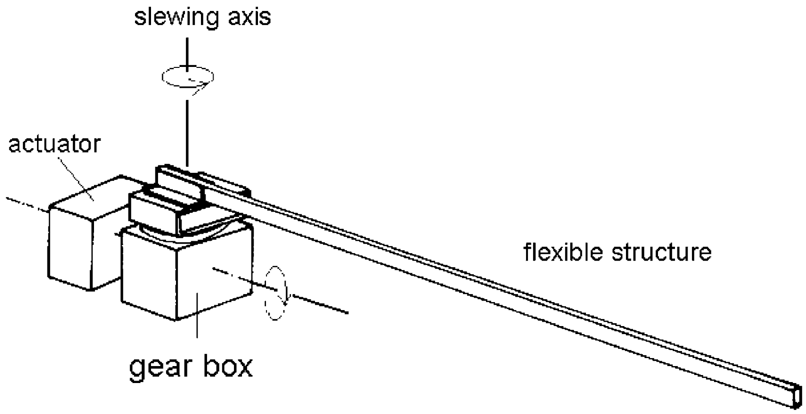

Fig. 1. Schematic of a slewing flexible structure and actuator system.

The dynamical system whose behavior is analyzed in this work consists of a dc motor (actuator) and a slewing flexible beam-like structure to be moved. A general schematic of this system is depicted in Fig. 1. Under the assumption of the ideal system approach, it is possible to prescribe the angular displacement of the motor axis, θ. In other words, its behavior is known beforehand and is some function of time. Otherwise, when a nonideal system approach is assumed, the voltage across the motor terminals, U, is allowed to be prescribed and the angular displacement, θ, is obtained through integration of the governing equations of motion of the energy source together with the governing equations of motion of the flexible structure. The behavior of the energy source can be affected by the flexible structure behavior through the coupling term represented by the reacting momentum M (see Fig. 1 and Eqs. (1)). In the latter case, the behavior of the slewing maneuver (θ) is known only when the whole set of governing equations is properly integrated.

In the modeling of flexible structures, there are some assumptions that can be made regarding its curvature. It is common to find in the literature the linear curvature approach (see, e.g. Refs. [3–5]. In this work, the nonlinear curvature approach is presented [6–10] and discussed.

The theory developed in this work can be applied to the study of lightweight structures such as solar panels in satellites and long and fast robotic manipulators.

2. Mathematical modelling

The nondimensional governing equations of motion for the nonideal system consisting of a slewing flexible structure (nonlinear curvature and two modes considered on the discretization, where q_1 and q_2 represent the modal amplitudes) moved by a limited energy source (dc motor) are given in the state form ($x_1 = q_1$; $x_2 = q_1$; $x_3 = q_2$; $x_4 = q_2$; $x_5 = \theta$; $x_6 = \theta$ and $x_7 = i_a$) in Refs. [6–10].

By using the same notation and values of Ref. [8] we easily obtain

$$
\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= \left(\frac{1}{AD-BC}\right)(Df_1 - Bf_2 + (BF-DE)(-g_3x_6 + g_4x_7 + Gx_1 + Hx_3)),\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= \left(\frac{1}{AD-BC}\right)(-Cf_1 + Af_2 + (CE-AF)(-g_3x_6 + g_4x_7 + Gx_1 + Hx_3)),\\
\dot{x}_5 &= x_6,\\
\dot{x}_6 &= -g_3x_6 + g_4x_7 + Gx_1 + Hx_3,\\
\dot{x}_7 &= -g_1x_7 - g_2x_6 + c_1U,
\end{aligned}
\tag{1}
$$

where

$$A = 1 + \in^2(c_{12}x_1^2 + c_{33}x_1x_3 + c_{18}x_3^2),\quad B = \in^2(c_{13}x_1^2 + c_{34}x_1x_3 + c_{19}x_3^2),$$

$$C = \in^2(d_{12}x_1^2 + d_{33}x_1x_3 + d_{18}x_3^2),\quad D = 1 + \in^2(d_{13}x_1^2 + d_{34}x_1x_3 + d_{19}x_3^2),$$

$$E = c_1 + \in^2(c_8x_1^2 + c_{28}x_1x_3 + c_{11}x_3^2),\quad F = d_1 + \in^2(d_8x_1^2 + d_{28}x_1x_3 + d_{11}x_3^2),$$

$$
\begin{aligned}
f_1 = {} & -w_1^2x_1 - \in^2(c_2x_8^2x_1 + c_3x_8^2x_3 - c_4x_8x_1x_2 - c_5x_8x_1x_4 - c_6x_8x_3x_2 - c_7x_8x_3x_4\\
& + c_{12}x_1x_2^2 + c_{29}x_1x_2x_4 + c_{15}x_1x_4^2 + c_{16}x_3x_2^2 + c_{30}x_2x_3x_4 + c_{19}x_3x_4^2 + c_{20}x_1^3\\
& + c_{31}x_1^2x_3 + c_{32}x_1x_3^2 + c_{27}x_3^3),
\end{aligned}
$$

$$
\begin{aligned}
f_2 = {} & -w_2^2x_3 - \in^2(d_2x_8^2x_1 + d_3x_8^2x_3 - d_4x_8x_1x_2 - d_5x_8x_1x_4 - d_6x_8x_3x_2 - d_7x_8x_3x_4\\
& + d_{12}x_1x_2^2 + d_{29}x_1x_2x_4 + d_{15}x_1x_4^2 + d_{16}x_3x_2^2 + d_{30}x_2x_3x_4 + d_{19}x_3x_4^2 + d_{20}x_1^3\\
& + d_{31}x_1^2x_3 + d_{32}x_1x_3^2 + d_{27}x_3^3),
\end{aligned}
$$

with boundary conditions $x_1(0) = 0$, $x_2(0) = 0$, $x_3(0) = 0$, $x_4(0) = 0$, $x_7(0) = 0$, $x_8(0) = 0$ and $x_9(0) = 0$. The reactive coupling momentum is given by $M = Gx_1 + Hx_3$.

In Eqs. (1), the first four equations are related to the dynamics of the flexible structure and the last three are related to the energy source (the first two of them related to the angular displacement, θ, and the last one related to the armature current in the dc motor, i_a).

All the following relations define all the coefficients in Eqs. (1) (see Ref. [8]):

$$T = \frac{1}{1.8780^2}\sqrt{\frac{\rho L^4}{EI}},\quad G = g_5\phi_1''(0),\quad H = g_5\phi_1''(0),\quad g_1 = \frac{R_aT}{L_m},\quad g_2 = N_g,$$

$$g_3 = \frac{N_g^2C_mT}{I_{\mathrm{shaft}} + N_g^2I_{\mathrm{motor}}},\quad g_4 = \frac{N_gK_tK_bT^2}{L_m(I_{\mathrm{shaft}} + N_g^2I_{\mathrm{motor}})},\quad g_5 = \frac{EIT^2}{L(I_{\mathrm{shaft}} + N_g^2I_{\mathrm{motor}})},$$

$c_1 = 0.5701570,\quad c_2 = -0.2851750,\quad c_3 = -1.7208800,\quad c_4 = -1.3361900,\quad c_5 = 47.1697000,$

$$c_6 = 49.6914000, \quad c_7 = -0.0762640, \quad c_8 = 0.9725700, \quad c_9 = -11.8426000, \quad c_{10} = -13.1035000,$$

$$c_{11} = 0.3021690, \quad c_{12} = 0.7803060, \quad c_{13} = -0.8595480, \quad c_{14} = 30.8545000, \quad c_{15} = -37.7213000,$$

$$c_{16} = 30.8550000, \quad c_{17} = -37.7213000, \quad c_{18} = 0.6784830, \quad c_{19} = -0.7643460,$$

$$c_{20} = 1.8259900 w_1^2 - 0.0560091, \quad c_{21} = -1.9865700 w_1^2, \quad -0.0219336,$$

$$c_{22} = -0.3290320 w_1^2 + 0.1494660, \quad c_{23} = 1.2589900 w_1^2 - 0.0352172,$$

$$c_{24} = 5.4885000 w_1^2 - 0.0116806, \quad c_{25} = -13.1073000 w_1^2 - 0.0110353,$$

$$c_{26} = 0.8074250 w_1^2 - 0.0560069, \quad c_{27} = -1.0981600 w_1^2 + 0.0226799,$$

$$c_{28} = c_9 + c_{10}, \quad c_{29} = c_{13} + c_{14}, \quad c_{30} = c_{17} + c_{18}, \quad c_{31} = c_{21} + c_{22} + c_{24}, \quad c_{32} = c_{23} + c_{25} + c_{26},$$

$$c_{33} = c_{14} + c_{16}, \quad c_{34} = c_{15} + c_{17}, \quad d_1 = 0.0906697, \quad d_2 = -1.1686000, \quad d_3 = -0.4543710,$$

$$d_4 = 0.8107680, \quad d_5 = -24.8756000, \quad d_6 = -24.6273000, \quad d_7 = 0.8949940, \quad d_8 = -0.4355640,$$

$$d_9 = 6.7840200, \quad d_{10} = 6.6598900, \quad d_{11} = -0.4958880, \quad d_{12} = -0.3491580, \quad d_{13} = 0.6370520,$$

$$d_{14} = -24.5398000, \quad d_{15} = 34.1991000, \quad d_{16} = -24.5398000, \quad d_{17} = 34.1995000,$$

$$d_{18} = -0.3721480, \quad d_{19} = 0.6663030, \quad d_{20} = -0.7527160 w_2^2 - 0.0913507,$$

$$d_{21} = 1.4485700 w_2^2 - 0.1260100, \quad d_{22} = 1.4789800 w_2^2 + 0.1876150,$$

$$d_{23} = -0.6607630 w_2^2 + 0.0986172, \quad d_{24} = 19.0692000 w_2^2 - 0.1049100,$$

$$d_{25} = -21.7338000w_2^2 - 0.0348521, \quad d_{26} = -1.0796100w_2^2 + 0.0412387,$$

$$d_{27} = 1.6448700w_2^2 - 0.0756208, \quad d_{28} = d_9 + d_{10}, \quad d_{29} = d_{13} + d_{14}, \quad d_{30} = d_{17} + d_{18},$$

$$d_{31} = d_{21} + d_{22} + d_{24}, \quad d_{32} = d_{23} + d_{25} + d_{26}, \quad d_{33} = d_{14} + d_{16}, \quad d_{34} = d_{15} + d_{17}.$$

In Eqs. (1), making $G = 0$ and $H = 0$, the nonideal system mathematical model reduces to the ideal system mathematical model, which is given by

$$\begin{aligned}
\dot{x}_1 &= x_2,\\
\dot{x}_2 &= \left(\frac{1}{AD - BC}\right)(Df_1 - Bf_2 + (BF - DE)(-g_3x_6 + g_4x_7)),\\
\dot{x}_3 &= x_4,\\
\dot{x}_4 &= \left(\frac{1}{AD - BC}\right)(-Cf_1 + Af_2 + (CE - AF)(-g_3x_6 + g_4x_7)),\\
\dot{x}_5 &= x_6,\\
\dot{x}_6 &= -g_3x_6 + g_4x_7,\\
\dot{x}_7 &= -g_1x_7 - g_2x_6 + c_1U.
\end{aligned} \tag{2}$$

3. System parameters and prescribed excitation

The set of first-order ordinary differential equations (1) and (2) are integrated through a predictor–corrector algorithm. The set of parameters for the flexible structure, the gear ratio values and the dc motor parameters considered for the numerical simulations are given by Table 1 [11,12].

The electric voltage, U, for the nonideal system approach, is prescribed according to the profile in Fig. 2. In the ideal system approach, the equations governing the dc motor and the equations governing the modal amplitudes, q_i, are decoupled and the first set of equations is used to

Table 1
Numerical values of the physical parameters

Flexible structure (and gear box)
Length of the beam, L: 0.3000 (m); height of the beam cross-section: 0.0050 (m); width of the beam cross-section: 0.0005 (m); structural damping, $\mu = 0.0000\,\mathrm{kg/ms}$; Young modulus (aluminum), $E = 0.7000 \times 10^{11}$ (N/m^2); gear ratio, N_g = variable (1,2 or 3);
DC motor
Maximum voltage across the motor terminals, $U_{\max} = 7.0000$ (V); viscous friction coefficient, $C_m = 0.0046$ (Nm s/rad); torque constant, $K_t = 0.0528$ (Nm/A); back e.m.f. constant, $K_b = 0.0528$ (Vs/rad); inductance, $L_m = 0.0031$ (H); armature resistance, $R_a = 1.9149(\Omega)$; moment of inertia of the rotor, $I_{\text{motor}} = 6.5400 \times 10^{-5}$ (kg m^2); inertia of the slewing axis, $I_{\text{shaft}} = 3.6900$ (kg m^2).

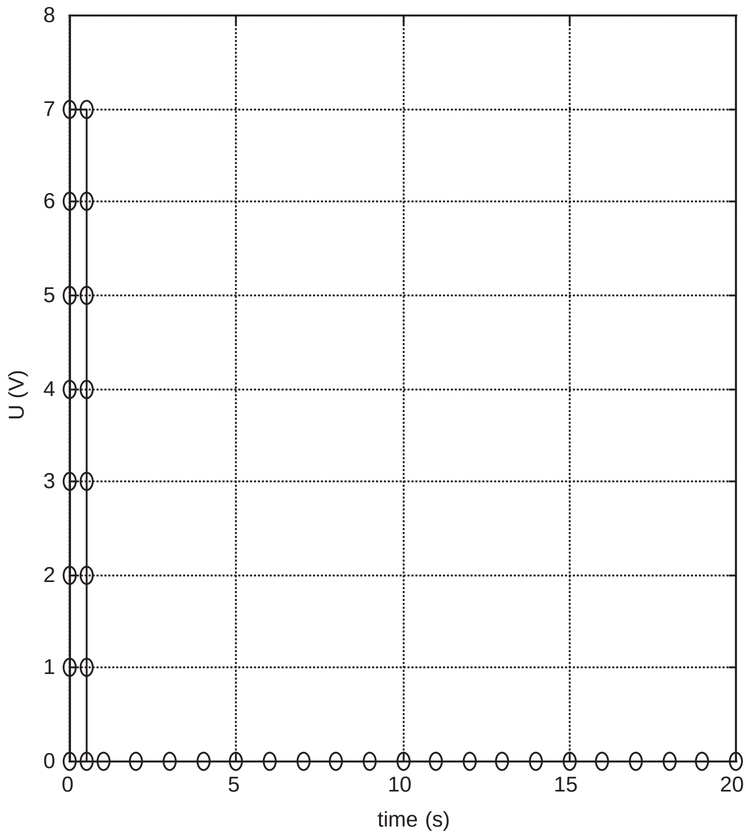

Fig. 2. Prescribed voltage, U.

prescribe the angular displacement and velocity, considering the same excitation (U) as in the nonideal case. The only kind of damping in the system responses will be the one provided by the interaction actuator–structure in cases where this interaction exists.

4. Numerical simulations

In the numerical simulations, which follow, the only system parameter that changes is the gear ratio, N_g. In doing so, one is varying the inertia of the subsystem actuator plus gearbox. The values of the gear ratio considered are 1 (no gearbox), 2 and 3. In all the cases presented here, for different levels of interaction between actuator and structure the ideal and the non-ideal approaches are compared. It is important to observe that the greater the value of N_g the smaller the before quoted interaction.

Among the results presented here, the ones shown in Fig. 3 illustrate the most meaningful interaction between the response of the flexible beam-like structure and the response of the actuator. The differences between the ideal system response (full line with small circles) and the nonideal system response (full line) are remarkable as one can see. These differences can be noted

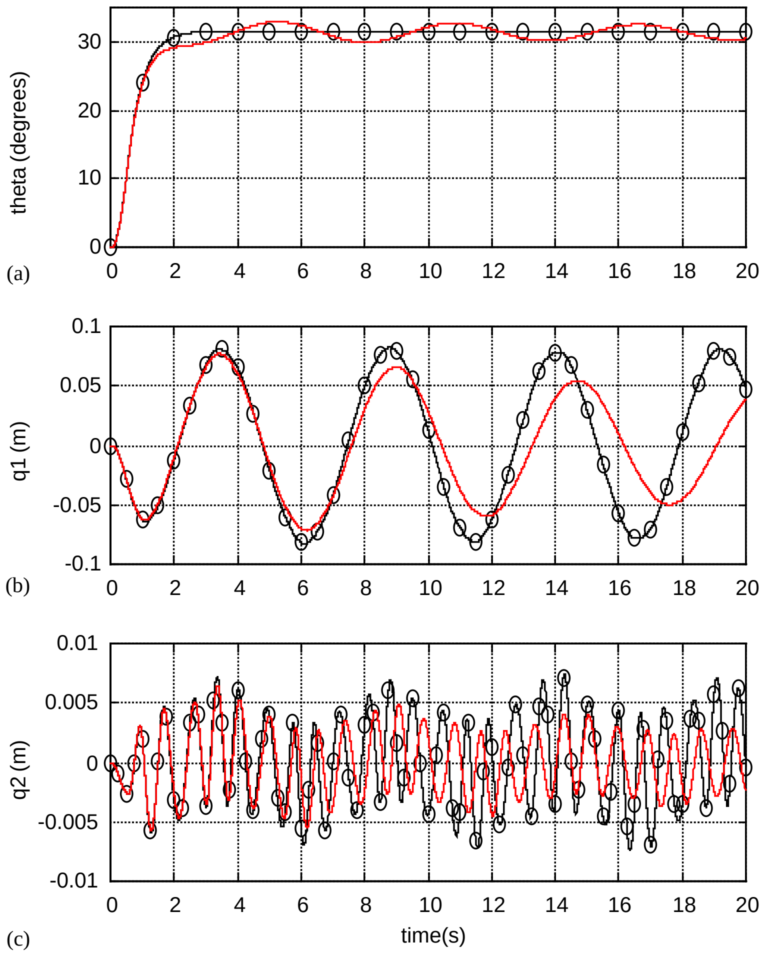

Fig. 3. Comparison between ideal system approach (-o-o-o-o-) and nonideal system approach (——) considering $N_g = 1$: (a) θ; (b) and (c) first and second mode solutions.

in the amplitudes as well as in the frequency. In the nonideal system approach, energy is allowed to flow between actuator and structure resulting in the decreasing of the amplitude of vibration of the beam. The greater the value of the beam reacting momentum, M, the greater this energy exchange. Under these circumstances, the inclusion of this kind of interaction in mathematical models for dynamical systems as the ones investigated here is a necessity.

According to Figs. 4 and 5, as the value of the gear ratio increases the influence of the flexible beam vibration on the dc motor response decreases. In Fig. 5, this influence is practically negligible and the same response is predicted for both the ideal system approach and the nonideal system approach.

The faster the slewing maneuver, the greater the influence of the nonlinearities (arising from the curvature assumptions) in the system response and its influence the source of excitation (through M). In this case, more critical are the system behavior more the care that must be taken in the modeling assumptions. Another aspect to be discussed regarding the increasing of the gear ratio

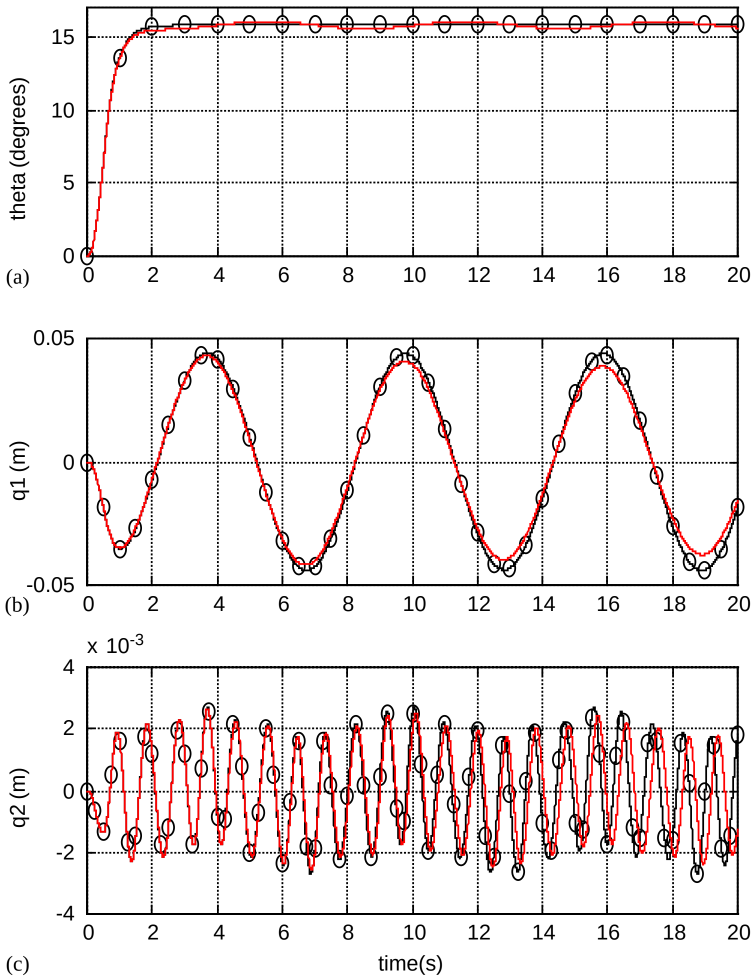

Fig. 4. Comparison between ideal system approach (-o-o-o-o-) and non-ideal system approach (——) considering $N_g = 2$: (a) θ; (b) and (c) first and second mode solutions.

values is that it makes the angular velocity of the slewing axis decrease, eliminating nonlinear effects.

Another fact that can be observed in Figs. 3–5 is the decrease in the amplitudes of vibration of the variables θ, q_1 and q_2 due to the damping effect resulting from the exchange of energy between the flexible beam-like structure and the energy source (dc motor). This effect of damping in the amplitudes of vibration was observed here only in the nonideal system responses and is remarkable in the situations where the actuator-structure interaction is sufficiently strong, as in the case where $N_g = 1$ (no gear box) in Fig. 3.

It is observed that the greater this interaction (as can be seen in the nonideal case in Fig. 3 compared with Fig. 5) the greater the damping effect. In the ideal system approach this damping effect is not observed and the energy is simply stored in the beam (there is no other kind of dissipation considered here). All the damping observed on the responses comes from the exchange

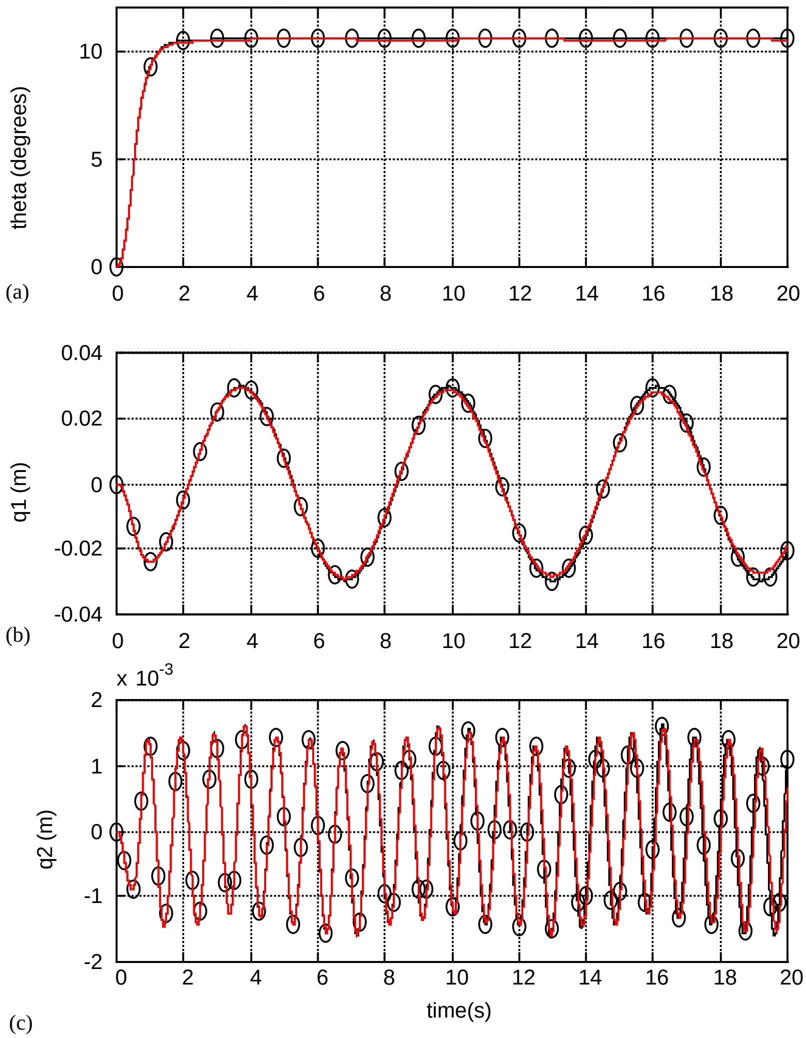

Fig. 5. Comparison between ideal system approach (-o-o-o-o-) and non-ideal system approach (——) considering $N_g = 3$: (a) θ; (b) and (c) first and second mode solutions.

of energy between actuator and structure. This fact represents an important result to be considered in the design of a control law for such systems.

In all the cases presented here, the influence of the second mode response is significant. The greater the angular velocity (and the smaller the gear ratio) the greater the influence of superior modes. It is noted for ideal and nonideal systems alike.

5. Conclusions

The mathematical modeling and numerical simulations presented in this work for a slewing nonlinear flexible beam-like structure with different gear box values between actuator and structure alert the necessity of modeling the interaction between the energy source and the moved system for some critical cases. It is also remarkable that this interaction causes a damping of the beam response.

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